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On the Complex Poisson's Ratio of a Urethane Rubber Compound

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Synopsis

The complex Poisson's ratio of a urethane rubber compound was determined for frequencies up to 700 cps. It is shown that the assumption made by earlier workers using this material, that Poisson's ratio is a numerical constant slightly less than 1/2, while approximately correct at low (creep) frequencies is definitely invalid in certain more elevated frequency bands.

Wave propagation in anelastic media is important to a variety of disciplines primarily concerned with mechanical wave radiation in real earth materials. Before propagation theories are applied to the uncontrolled earth, however, it is desirable to test them under known conditions in the laboratory. Since the results of such theories are frequently expressed as Fourier integrals it is necessary that the properties of the laboratory test material be known over a wide frequency range. Even for an isotropic material subjected to only infinitesimal strains two functions are required to specify the rheological properties. A moderately large volume of literature is available describing the measurement of some one modulus for a variety of materials, but the number of anelastic substances for which two moduli have been measured as functions of frequency is very small. The most frequent practice has been to measure one modulus and make some convenient assumption about the second, such as incompressibility.

Using Bland's[1] theory of linear viscoelasticity one is led to correspondence principles between elastic constants and viscoelastic moduli enabling one to write the complex Poisson's ratio $\nu(f)$ in the same form as for the elastic case, namely

$$\nu(f) = \frac{3K(f) - 2\mu(f)}{6K(f) + 2\mu(f)} \tag{1}$$

where $K(f)$ is the complex bulk modulus and $\mu(f)$ is the complex shear modulus. For materials approximately incompressible at any frequency f then $K(f)$ is a large number. On expanding the right member of eq. (1) in powers of $\mu/3K$ by the binomial theorem, one is then led to (see following page)

$$\nu(f) \doteq \frac{1}{2} - \frac{\mu(f)}{2K(f)} \tag{2}$$

or in the limit of complete incompressibility

$$\lim_{k\to\infty} \nu(f) = 1/2 \tag{3}$$

Any substance within its linearly viscoelastic range and for frequencies at which its bulk modulus is much larger than its shear modulus should then display a Poisson's ratio which should be slightly less than 1/2.

A particular urethane rubber compound referenced in the literature variously under Hysol numbers 4485, 2085, and 3562, and 8705 (depending on the form in which it was purchased and on the date) has been the object of considerable experiment designed to determine its physical properties. Dally[2] used a double pendulum technique to measure Young's modulus as a function of loading rate. In order to reduce his experimental data he made the assumption that the test material was incompressible. Arenz,[3] using the time–temperature shifting principle on the data from a constant strain rate tester, developed Young's modulus as a function of time over the remarkable range from 10^2 to 10^{-16} min. In order to complete the material property specification Poisson's ratio was assumed to be a numerical constant independent of frequency.

In order to test the incompressibility and other physical property hypotheses made about this particular urethane rubber a test program was sponsored in several laboratories where suitable test equipment existed and Young's modulus and/or the shear modulus measured over various frequency bands and at various temperatures. Baltrukonis and Blömquist[4] measured the shear modulus as a function of temperature and frequency by means of torsional pendula enclosed in an environmental chamber. Cunningham, Brown, and Johnson[5] measured the dynamic Young's modulus as a function of frequency at room temperature. Vieira. and Riley[6] performed creep tests, complex Young's modulus, and complex shear modulus tests in an environment maintained at 70°F. and 40% humidity. The reports on these various tests have been collected into a compendium,[7] the data from which are herewith used to evaluate the incompressibility hypothesis.

Poisson's ratio $\nu(f)$ was calculated from

$$\nu(f) = \left\{\frac{E'\mu' + E''\mu''}{2[(\mu')^2 + (\mu'')^2]}\right\} + i\left\{\frac{E''\mu' - E'\mu''}{2[(\mu')^2 + (\mu'')^2]}\right\} \tag{4}$$

where the first and second terms of the right member of eq. (4) are respectively the real and imaginary parts of the complex Poisson's ratio and $E' = E'(f)$ = real part of Young's modulus, $E'' = E''(f)$ = imaginary part of Young's modulus, $\mu' = \mu'(f)$ = real part of shear modulus, $\mu'' = \mu''(f)$ = imaginary part of shear modulus, and $i = \sqrt{-1}$.

The results of the calculations based on eq. (4) are graphed in Figure 1. The Baltrukonis and Blömquist[4] (BB) shear modulus data used for the

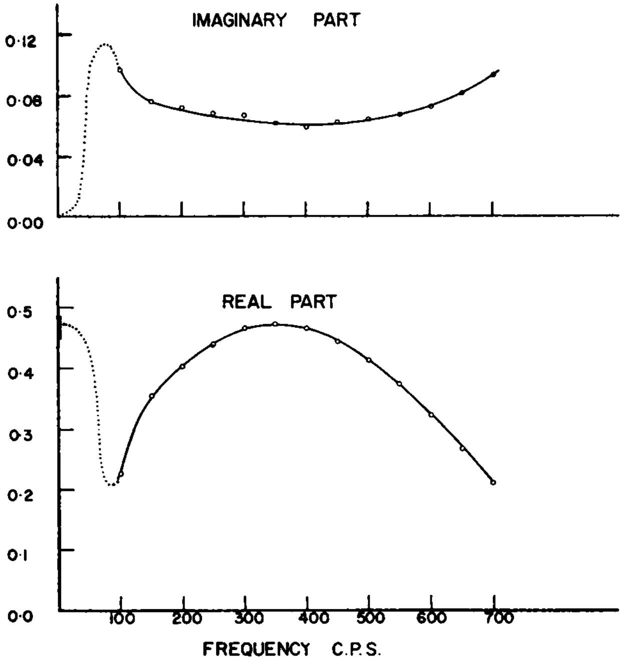

Fig. 1. Complex Poisson's ratio.

above calculations included only those measurements made in an environment of 70°F.; data taken at other temperatures and time temperature shifted were not used. Cunningham, Brown, and Johnson's[5] (CBJ) dynamic Young's modulus data were adapted by converting to real and imaginary parts, as were the data of Vieira and Riley[6] (VR). Real and imaginary parts of Young's modulus and the shear modulus for all investigators were plotted and a smooth curve fitted to the data by eye in the frequency range 100–700 cps. By using the fitted curves, values of $E'(f)$, $E''(f)$, $\mu'(f)$ and $\mu''(f)$ at 50 cps intervals were picked off, $\nu'(f)$ and $\nu''(f)$ calculated, and the curve shown in Figure 1 plotted. The VR creep test results from which Poisson's ratio was calculated are also shown plotted at slightly more than zero frequency as a bar the length corresponding to the range of values of ν observed with observations made in the time interval 10^{-1}–10^{3} min.

On inspecting the Poisson's ratio curves and points of Figure 1 it is apparent that Poisson's ratio cannot be considered a numerical constant at all frequencies for Hysol 4485 in the room temperature regime. In particular, if waveforms are to be accurately predicted in this material for wave packets containing frequencies in the region above creep frequencies and below 200 cps or at frequencies above and near 500 cps, allowance must be made for the functional variation of Poisson's ratio with frequency. The Poisson's ratio values shown dotted in the region above creep frequencies but below

100 cps are interpolations, and further experimental work is required for accurate specification of Poisson's ratio in this frequency band.

References

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Résumé

Le rapport complexe de Poisson d'un composé caoutchouteux à base d'uréthanne a été déterminé pour des fréquences allant jusqu'à 700 c.p.s. On a montré que l'hypothèse faite par des chercheurs antérieurs utilisant ce matériel, suivant laquelle le rapport de Poisson est une constante numérique légèrement inférieure à 0.5 est absolument non-applicable dans les bandes de fréquence plus élevées alors qu'elle est approximativement correcte aux faibles fréquences.

Zusammenfassung

Das komplexe Poissonverhältnis eines gefüllen, Urethankautschuks wurde bei Frequenzen bis 700 hz bestimmt. Es wird gezeigt, dass die Annahme früherer Autoren, dass das Poissonverhältnis dieses Stoffes eine numerische Konstante schwach un erhalb 0.5 ist, zwar für niedere Frequenzen (Kriechen) angenähert zutrifft, jedoch mit Sicherheit bei gewissen höheren Frequenzen ungültig ist.

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